The Model and the Parameters

1 Generator model

The generator model used for the simulation is a 3-dimensional model with the state variables of the machine angle δ(rad), the machine rotation speed deviation ω(p.u.) and the voltage behind the transient reactance e'_q(p.u.). The dots denote differentiation with respect to time.

$$\dot{\delta} = \omega_B \, \omega \tag{1}$$

$$\dot{\omega} = \frac{1}{M}(P_m - P_e - D\omega) \tag{2}$$

$$\dot{e'_q} = \frac{1}{T'_{do}} \left\{ -e'_q - (X_d - X'_d) i_d + e_f \right\} \tag{3}$$

$$i_d = -c_1 V_\infty \sin\delta - c_2 V_\infty \cos\delta + c_3 \, e'_q \tag{4}$$

$$i_q = c_4 V_\infty \sin\delta - c_5 V_\infty \cos\delta + c_6 \, e'_q \tag{5}$$

$$P_e = \left\{ e'_q + (X_q - X'_d) i_d \right\} i_q \tag{6}$$

$$e_d = X_q i_q, \qquad e_q = e'_q - X'_d i_d \tag{7}$$

$$V_t = \sqrt{{e_d}^2 + {e_q}^2} \tag{8}$$

Values of the parameters $c_1, c_2, \ldots, c_6$ in Eq.(4), Eq.(5) are defined with the parameters a_1, a_2, a_3, a_4 whose values are determined by the external impedance. Line fault simulation is done by changing the values of $c_1, c_2, \ldots, c_6$ according to the phase of the fault sequence, which is described in detail in the next section.

2 External impedace model for the 3-phase short circuit

For the simulating the 3-phase short circuit, sigle-phase equivalent circuits of the external impedance for four cases of prefault period, during fault, post-clearance period (during one-line transmission) and post-reclosure period are derived. The dynamics of the transmission line circuit is ignored and the phasor expression (denoted by bold letters) is used.

Figure 1 shows the transmission line model. In the figure, $\boldsymbol{Z_l}$ is the series impedace of the line, $\boldsymbol{Z_t}$ is the series impedace of the transformer (the leakage reactance and the resistance), a indicates the ratio of the two-line part for $\boldsymbol{Z_l}$ b indicates the position of the 3-phase short circuit fault on the two-line part, $\boldsymbol{E_t}$ is the generator terminal phase voltage, $\boldsymbol{E_\infty}$ is the infinite bus phase voltage and $\boldsymbol{I}$ is the generator phase current. The switches of S_1, S_2 virtually express the line fault.

2.1 Equivalent circuit before fault

In the prefault period, S_1 is open and S_2 is closed in Figure 1, so the single-phase equivalent circuit becomes as Figure 2. Therefore,

$$\boldsymbol{E_t} = \boldsymbol{E_\infty} + (\boldsymbol{Z_t} + \boldsymbol{Z_l})\boldsymbol{I} \; . \tag{9}$$

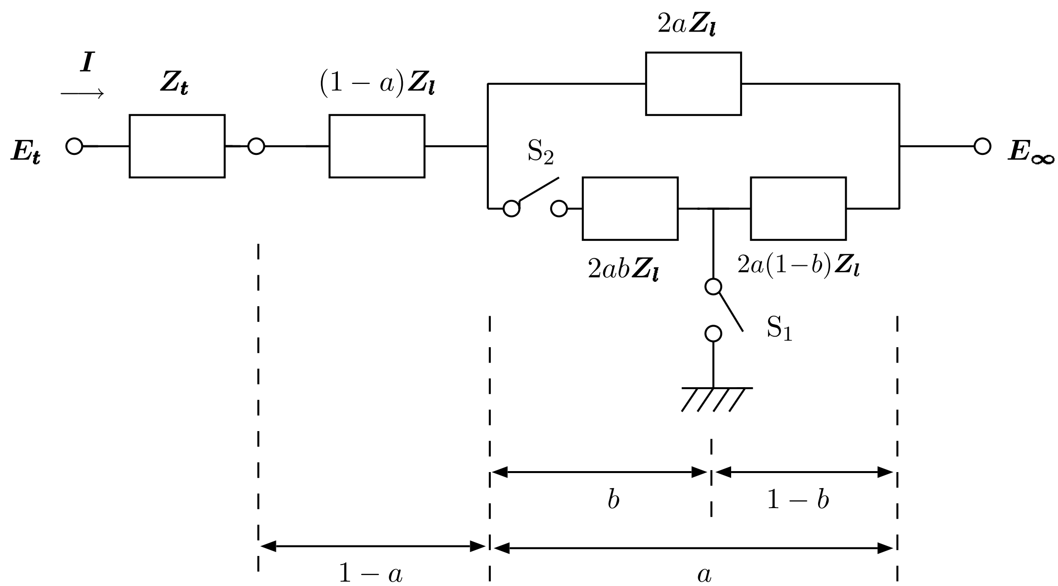

Figure 1: External impedance

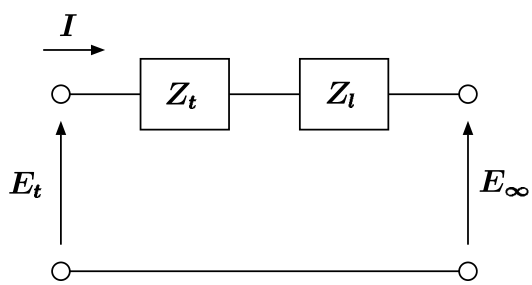

Figure 2: Prefault equivalent circuit

2.2 Equivalent circuit during fault

During the 3-phase short circuit fault, S_1 is closed and S_2 is open in Figure 1. The single-phase equivalent circuit becomes as Figure 3. If the π-circuit in the right of Figure 3 is converted to the T-circuit, it becomes like Figure 4. Because Figure 4 is a T-circuit, Eq.(10) is obtained using the formula of the hybird matrix for T-circuits.

$$\begin{pmatrix} \boldsymbol{E_t} \\ \boldsymbol{I'} \end{pmatrix} = \begin{pmatrix} \boldsymbol{Z_t} + \dfrac{1-a+b+ab}{1+b}\boldsymbol{Z_l} & \dfrac{b}{1+b} \\ -\dfrac{b}{1+b} & \dfrac{1}{a(1-b^2)\boldsymbol{Z_l}} \end{pmatrix} \begin{pmatrix} \boldsymbol{I} \\ \boldsymbol{E_\infty} \end{pmatrix} \tag{10}$$

Therefore,

$$\boldsymbol{E_t} = \frac{b}{1+b}\boldsymbol{E_\infty} + \left(\boldsymbol{Z_t} + \frac{1-a+b+ab}{1+b}\boldsymbol{Z_l}\right)\boldsymbol{I}\ . \tag{11}$$

As a result, the single-phase equivalent circuit becomes as Figure 5. The generator can be assumed to be conneted to the infinite bus whose voltage is multiplied by $b/(1+b)$ through the impedance of $\boldsymbol{Z_t} + \boldsymbol{Z_l}(1-a+b+ab)/(1+b)$.

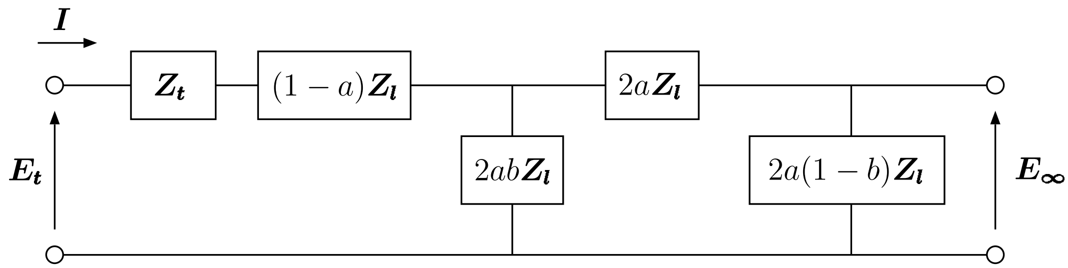

Figure 3: Equivalent circuit #1 during fault

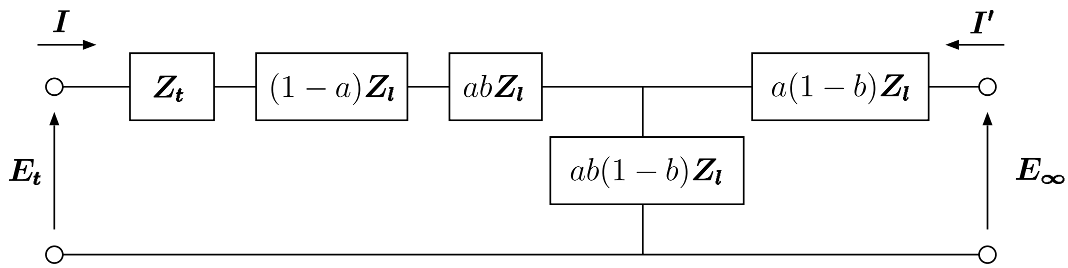

Figure 4: Equivalent circuit #2 during fault

2.3 Equivalent circuit after the fault is cleared

During the one-line transmission (after the fault clerance) S_2 is open in Figure 1, so the single-phase equivalent circuit is like Figure 6 and

$$\boldsymbol{E_t} = \boldsymbol{E_\infty} + \{\boldsymbol{Z_t} + (1+a)\boldsymbol{Z_l}\}\boldsymbol{I} \ . \tag{12}$$

2.4 Equivalent circuit after reclosure

After reclosure of the line, the circuit again becomes identical as in the prefault period and Eq. (9) holds.

2.5 Parameter value variation in the fault sequence

All the cases above are summarized in this section. Let the relationship between the generator terminal phase voltage, the infinite bus phase voltage and the generator phase current as

$$\boldsymbol{E_t} = (a_1 + \mathrm{j}a_2)\boldsymbol{E_\infty} + (a_3 + \mathrm{j}a_4)\boldsymbol{I} \ , \tag{13}$$

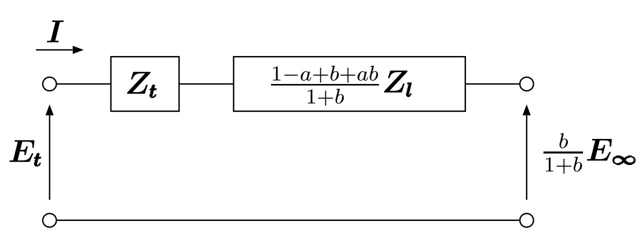

Figure 5: Equivalent circuit #3 during fault

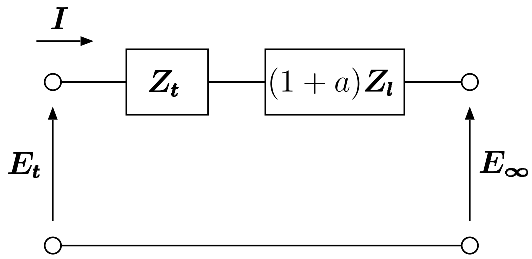

Figure 6: Equivalent circuit after fault clearance

Table 1: Parameter value variation for line fault sequence

	t: time from fault ocuurence	S_1	S_2	a_1	a_2	a_3	a_4
prefault	$t < 0$	OFF	ON	1	0	R_l	$X_t + X_l$
during fault	$0 \leq t < \tau_{fd}$	ON	ON	$b/(1+b)$	0	$k_r R_l$	$X_t + k_r X_l$
post-clearance	$\tau_{fd} \leq t < \tau_{fd} + \tau_{ri}$	OFF	OFF	1	0	$R_l(1+a)$	$X_t + X_l(1+a)$
post-reclosure	$\tau_{fd} + \tau_{ri} \leq t$	OFF	ON	1	0	R_l	$X_t + X_l$

where $k_r = (1 - a + b + ab)/(1+b)$

i.e.,

$$e_d = a_1 e_{d\infty} - a_2 e_{q\infty} + a_3 i_d - a_4 i_q \quad (14)$$

$$e_q = a_1 e_{q\infty} + a_2 e_{d\infty} + a_3 i_q + a_4 i_d \quad (15)$$

and the values of a_1, a_2, a_3, a_4 at the t (s) after the fault occurence are as in Table 1. Note that the resitance at the transformer is neglected and

$$\boldsymbol{Z_t} = \mathrm{j}X_t, \quad \boldsymbol{Z_l} = R_l + \mathrm{j}X_l \ . \quad (16)$$

τ_{fd} indicates the fault duration time (s) and τ_{ri} indicates the interval (s) from the fault clearance until the line reclosure.

$e_{d\infty}$(p.u.) and $e_{q\infty}$(p.u.) are expressed as

$$e_{d\infty} = V_\infty \sin\delta, \quad e_{q\infty} = V_\infty \cos\delta \quad (17)$$

Using the parameters a_1, a_2, a_3, a_4, the values of the parameters $c_1, c_2, \ldots, c_6$ in Eq.(4) and Eq.(5) are obtained as

$$c_1 = \frac{a_1 a_3 + a_2(X_q + a_4)}{c_0} \quad (18)$$

$$c_2 = \frac{a_1(X_q + a_4) - a_2 a_3}{c_0} \quad (19)$$

$$c_3 = \frac{X_q + a_4}{c_0} \quad (20)$$

$$c_4 = \frac{a_1({X_d}' + a_4) - a_2 a_3}{c_0} \quad (21)$$

$$c_5 = \frac{a_1 a_3 + a_2({X_d}' + a_4)}{c_0} \quad (22)$$

$$c_6 = \frac{a_3}{c_0} \quad (23)$$

where

$$c_0 = a_3^2 + (X_q + a_4)(X_d{}' + a_4)\ . \tag{24}$$

Eq.(18)-(23) are derived by solving Eq.(14), Eq.(15) and Eq.(7) simultaneously.

3 Parameter Values

Table 2 shows the default parameter values used for the simulation.

Table 2: Constants used in simulation

X_d	1.030 p.u.
X_q	0.618 p.u.
ω_B	120π rad/s
V_∞	1.0 p.u.
X_t	0.1 p.u.
X_l	0.37 p.u.
R_l	0.073 p.u.
M	10.59 s
T'_{do}	7.61 s
X'_d	0.326 p.u.
D	3
e_f	1.16 p.u.
P_m	0.8 p.u.
a	0.9
b	0.1
τ_{fd}	4/60 s